Stat 155 Lecture 13 Notes

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1 Evolutionary Game Theory

1.1 Criticisms of Nash equilibria

What's wrong with Nash equilibria? There are many criticisms one might have:

- Will all players know everyone's utilities?
- Maximizing expected utility does not (explicitly) model risk aversion.
- Will players maximize utility and completely ignore the impact on other players' utilities?
- How can the players find a Nash equilibrium?
- How can the players agree on a Nash equilibrium to play?
- Will players actually randomize?

We will discuss some alternative equilibrium concepts:

1. Correlated equilibrium
2. Evolutionary stability
3. Equilibria in perturbed games

1.2 Evolutionarily stable strategies

Say there is a population of individuals. There is a game played between randomly chosen pairs of individuals, where each individual has a pure strategy encoded in its genes. A higher payoff gives higher reproductive success. This can push the population towards stable mixed strategies.

Consider a two-player game with payoff matrices A, B. Suppose that it is symmetric ($A = B^\top$). Consider a mixed strategy x. Think of x as the proportion of each pure strategy in the population.

Suppose that x is invaded by a small population of mutants z (that is, playing strategy z). The criteria for x to be an evolutionary stable strategy will imply that, for small enough ε, the average payoff for xs will be strictly greater than that for zs, so the invaders will disappear. Will the mix x survive? Say a player who plays x goes against an invader. Then the expected payoff is $x^\top Az$. If, instead, a player with strategy x goes against another one with strategy x, then the expected payoff is $x^\top Ax$. Since $1-\varepsilon$ is the proportion of players with strategy x, and ε is the proportion of players with strategy z, the utility of a player with strategy x is

$$(1-\varepsilon)x^\top Ax + \varepsilon x^\top Az = x^\top A((1-\varepsilon)x + \varepsilon z).$$

Similarly, the utility for an invader is

$$(1-\varepsilon)z^\top Ax + \varepsilon z^\top Az = z^\top A((1-\varepsilon)x + \varepsilon z).$$

Definition 1.1. A mixed strategy $x \in \Delta_n$ is an *evolutionarily stable strategy* (ESS) if, for any pure strategy z,

1. $z^\top Ax \leq x^\top Ax$ ((x, x) is a Nash equilibrium).
2. If $z^\top Ax = x^\top x$, then $z^\top Az < x^\top Az$.

1.3 Examples of strategies within populations

Example 1.1. Two players play a game of Hawks and Doves for a prize of value $v > 0$. They confront each other, and each chooses (simultaneously) to fight or to flee; these two strategies are called the "hawk" (H) and the "dove" (D) strategies, respectively. If they both choose to fight (two hawks), then each incurs a cost c, and the winner (either is equally likely) takes the prize. If a hawk faces a dove, the dove flees, and the hawk takes the prize. If two doves meet, they split the prize equally.

The payoff bimatrix is

	H	D
H	$(v/2-c, v/2-c)$	$(v, 0)$
D	$(0, v)$	$(v/2, v/2)$

If, for example, we set $v = c = 2$, we get the payoff bimatrix The payoff bimatrix is

	H	D
H	$(-1, -1)$	$(2, 0)$
D	$(0, 2)$	$(1, 1)$

The pair (x, x) with $x = (1/2, 1/2)$ is a Nash equilibrium. Is it an evolutionarily stable strategy? Consider a mutant pure strategy z. We have $z^\top Ax \le x^\top Ax$ because (x, x) is a Nash equilibrium. If $z^\top Ax = z^\top Ax$, then is $z^\top Az < x^\top Az$? For $z = (1, 0)$ (that is, H)

$$z^\top Az = -1 < -1/2 = x^\top Az.$$

For $z = (0, 1)$ (that is, D)

$$z^\top Az = 1 < 3/2 = x^\top Az.$$

So x is an ESS.

Example 1.2. Consider a game of rock-paper-scissors. The payoff matrix for Player 1 is

	R	P	S
R	0	-1	1
P	1	0	-1
S	-1	1	0

The pair (x, x) with $x = (1/3, 1/3, 1/3)$ is a Nash equilibrium. Is it an ESS? We need to check that if $z^\top Ax = x^\top Ax$ then $z^\top Az < x^\top Az$. But for any pure strategy z, $z^\top Ax = 0 = z^\top Az$. So x is not an ESS.

The example of rock-paper-scissors shows us that cycles can occur, with the population shifting between strategies. This actually happens in nature.

Example 1.3. The males of the Uta Stansburiana lizard come in three colors. The colors correspond to different behaviors, which allow them to attract female mates:

1. Orange throat (aggressive, large harems, defeats blue throat)

2. Blue throat (less aggressive, small harems defeats yellow striped)

3. Yellow striped (submissive, look like females, defeats orange throat[1])

In nature, there is a 6 year cycle of shifting population proportions between these three colors.

[1] The yellow-striped lizards sneak into the territory of the orange throats and woo away the females.